

# Rational Exponents and Radicals Notes

## Rational Exponent

$$x^{\frac{m}{n}} = \sqrt[n]{x^m}$$

## Laws of Radicals

$$\left(\sqrt[n]{a}\right)^n = a, a > 0$$

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}, a, b > 0$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, a, b > 0$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$$

**Sample Problem 1:** Change the following expression in rational exponent to radicals.

1.  $a^{\frac{3}{4}} = \sqrt[4]{a^3}$

2.  $3^{\frac{1}{2}} = \sqrt{3}$

**Sample Problem 2:** Change the following radicals to rational exponent.

3.  $\sqrt{5} = 5^{\frac{1}{2}}$

4.  $\sqrt[4]{a^3} = a^{\frac{3}{4}}$

**Sample Problem 3:** Simplify the following expression in rational exponent.

5.  $(x^3)^{\frac{2}{3}} = \sqrt[3]{(x^3)^2} = \sqrt[3]{x^6} = x^2$

6.  $(a^2b^4)^{\frac{3}{2}} = \sqrt{(a^2b^4)^3} = \sqrt{a^6b^{12}} = a^3b^6$

7.  $25^{\frac{3}{2}} = \sqrt{25^3} = \sqrt{(5^2)^3} = \sqrt{5^6} = 5^3 = 125$

8.  $4^{-\frac{5}{2}} = \frac{1}{4^{\frac{5}{2}}} = \frac{1}{\sqrt{4^5}} = \frac{1}{\sqrt{(2^2)^5}} = \frac{1}{\sqrt{2^{10}}} = \frac{1}{2^5} = \frac{1}{32}$

**Sample Problem 4:** Evaluate the following radicals.

9.  $\left(\sqrt{x^3}\right)^2 = \left(x^{\frac{3}{2}}\right)^2 = x^{\frac{6}{2}} = x^3$

10.  $\sqrt[6]{4} = \sqrt[3]{\sqrt{4}} = \sqrt[3]{2}$

11.  $\sqrt[6]{4y^4} = \left(2^2y^4\right)^{\frac{1}{6}} = (2y^2)^{\frac{1}{3}} = \sqrt[3]{2y^2}$

12.  $\sqrt[3]{\frac{-2}{8x^6}} = \frac{(2)^{\frac{1}{3}}}{(-2^3x^6)^{\frac{1}{3}}} = \frac{\sqrt[3]{2}}{-2x^2}$